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Effect of Flow Discontinuities on Oscillatory Pressures Measured in Supersonic Wind Tunnels

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In 1959, Mollø-Christensen and Martuccelli[1] pointed out that surface-pressure measurements on an oscillating model in a supersonic wind tunnel may be greatly affected by the presence of possible wind-tunnel flow discontinuities. They further suggested that this fact may be responsible for the lack of any published experimental data on supersonic pressure distributions on oscillating wings. This situation still exists today. Therefore, it may be appropriate to present some new evidence, which indicates that the effect of flow discontinuities does not always have to be very large, and which suggests a practical method for assessing the magnitude of the effect and to determine suitable corrections if any are required.

Pressure interference associated with tunnel-flow discontinuities usually is very small as compared to steady surface pressures, but may become significant when compared to the much smaller oscillatory surface pressures. This interference effect is caused primarily by the fact that, in general, a point on the surface of an oscillating model continuously changes its location in the wind tunnel. Thus, the measured surface pressure at that point is a function of both the instantaneous mean flow conditions as given by the attitude and motion of the model and of any local flow discontinuities which may be present along the path of the point of measurement. To separate these two effects, an experimental condition must be found where only one of them is present or else where they are both present but in a known combination. The method described here requires a separate experiment during which the model performs additional oscillation in such a way that the mean flow conditions remain practically constant while the point of measurement traverses the same flow discontinuities as those intersected during the primary experiment. In this way, the aerodynamic effect is rendered negligible and the additional experiment gives only the interference effect to be determined.

Table 1 Experimental and theoretical values of oscillatory pressure ratio P_{osc}/P_0

	α_{mean}	P_{osc}/P_0 Pitching oscillation $\Delta\theta = 1.5°$	P_{osc}/P_0 Longitudinal oscillation $\Delta x = 0.019\,c$
Experiment, sidewall	0.14°	0.0056	0.0003
Experiment, topwall	0.30°	0.0058	0.0001
Experiment, topwall	−0.94°	0.0053	...
Linear theory	0	0.010	...
Nonlinear theory	0	0.005	...

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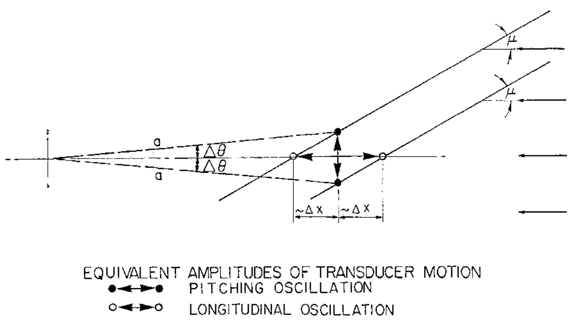

Fig. 1 Determination of equivalent amplitude of longitudinal oscillation; refraction across bow shock wave neglected.

If the primary experiment is an oscillation-in-pitch, the additional experiment may be a translational oscillation in the longitudinal direction‡ of the model, with suitably adjusted amplitude. If the flow discontinuities are assumed to be weak enough to propagate as Mach waves (a reasonable assumption in a good wind tunnel) and if the refraction of the flow discontinuities through the bow shock wave of the model is neglected, the required amplitude Δx of the longitudinal oscillation may be obtained from

$$\Delta x = a(\Delta\theta)\cot\mu \qquad (\Delta\theta \ll 1) \tag{1}$$

where $\mu = \sin^{-1}(1/M)$ is Mach angle, a is the distance of the point of measurement from the axis of oscillation and $\Delta\theta$ is the amplitude of the pitching oscillation (see Fig. 1).

The effect on Δx of refraction through a moving bow shock wave can be shown to be small for relatively weak bow shocks (e.g., such as on wings with sharp leading edge and relatively small nose angle) and small amplitudes. For instance, in the present experiments, this effect was of the order of 5% and could be neglected. In cases where the foregoing conditions are no longer met, a more detailed calculation of Δx may be required with refraction taken into account. This may be particularly important when the effects of flow discontinuities are large and irregular and when the point of measurement is far downstream from the leading edge.

Oscillatory pressure measurements were performed on a half model of an aspect ratio 3 cropped delta wing with taper ratio 0.072 and 5% thick biconvex profile in the Mach 1.8 nozzle of the NAE 30-in. × 16-in. suction wind tunnel. The model was mounted either on the flat sidewall opposite to the single nozzle wall of the tunnel or on the topwall of the tunnel. The model was oscillated in pitch with an amplitude of ±1.5° around an axis 0.45 c behind the wing apex, where c is the wing root chord. In addition, it was also oscillated (at slightly different frequency) in longitudinal translation, with the amplitude determined by means of Eq. (1). The quantities measured are shown in Fig. 2, where trace 1 is model deflection in pitch, traces 2 and 3 are oscillatory surface pressures in pitching and longitudinal oscillations, respectively, and trace 4 is aerodynamic and electronic noise measured

‡ At hypersonic speeds, it may be better to perform the additional experiment as a slow plunging oscillation, since otherwise the required amplitude could become impractically large.

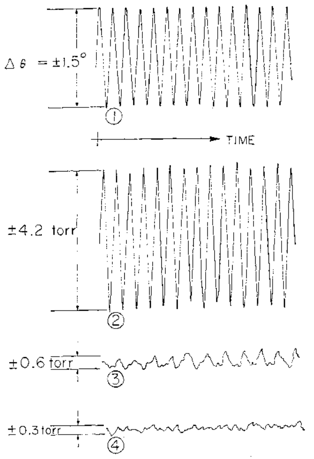

Fig. 2 Sample oscillograph records.

with wind on but no oscillation. Details of the experiments and more complete results are available in Ref. 2.

The experimental (after subtracting noise) and theoretical ratios of the amplitude P_{osc} of the oscillatory pressure to stagnation pressure P_0 at a station 0.93 c behind the wing apex and 0.23 c outboard of the wing root are given in Table 1. The effect of a slight nonzero mean incidence on the measured oscillatory pressure was small, as can be seen by comparing rows 2 and 3 in Table 1. Hence the value in rows 1 and 2 may be compared to the theoretical values which were calculated for zero mean wind incidence. Such theoretical values were obtained by private communication from J. J. Kacprzyński and were based on Ref. 3.

As may be seen, the oscillatory pressure ratio P_{osc}/P_0 for longitudinal oscillation is only of the order of 2–6% of the corresponding value for pitching oscillation. Thus, the effect of wind-tunnel flow discontinuities in this particular case appears to be relatively small. This conclusion is further corroborated by the fact that the results obtained with the model mounted on the sidewall and those obtained with the model mounted on the topwall (i.e., in a completely different local flow environment) agree within 4%.

In cases where the effect of flow discontinuities is not small, the longitudinal oscillation may be used to determine suitable corrections. A recording should be made of the oscillatory pressure for both modes of oscillation vs the motion of the wing. The oscillatory pressure for the pitching oscillation should then be corrected using values obtained for longitudinal oscillation for corresponding locations of the wing station with respect to flow discontinuities. In practice, the frequencies of the two oscillations may be different and, given nearly sinusoidal pressure variations and low reduced frequencies (small phase shifts), the amplitudes of the two oscillatory pressures may be directly superimposed.

It also may be seen from Table 1 that the experimental results, at least for this particular wing station, seem to agree much better with the nonlinear theory than with the linear one. As a further check of the experimental procedure, the measured oscillatory pressure was compared with a quasi-steady prediction based on the measured mean pressure at two different wing incidences. It was found that, within the accuracy of the present measurements, the oscillatory pressure could indeed be considered as quasi-steady, as would be expected for the relatively low reduced frequency of the experiment ($k = 0.06$). It may be noted that, in general, the small oscillatory pressure component can be measured with higher absolute accuracy than the relatively larger steady pressure component. Thus, a series of pressure measurements on a fixed wing in several successive longitudinal locations would not be as practical as the method described here.

In summary then, the effect of wind-tunnel flow discontinuities, which previously was considered to be a serious obstacle in measuring oscillatory surface pressures at supersonic speeds, was found to be relatively small, at least for the nozzle and for the wind tunnel used for this investigation. An experimental method, employing longitudinal oscillation of the model, was suggested for assessing the magnitude of this effect and for determining, if necessary, the required corrections. The measured oscillatory pressure in the vicinity of the trailing edge of the wing agreed well with the prediction of an unsteady nonlinear supersonic theory.

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Buckling of Clamped Skew Plates

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Nomenclature

a, b	= dimensions of plate, see Fig. 1
D	= $Eh^3/12(1 - \nu^2)$, flexural rigidity of the plate
h	= plate thickness
m, n, r, s	= indices in the deflection series
M	= maximum value of indices m, r
N	= maximum value of indices n, s
N_x, N_{xy}, N_y	= midplane forces per unit length, Fig. 1
$\bar{R}_x, \bar{R}_{xy}, \bar{R}_y$	= nondimensional midplane force parameters, N_xb^2/π^2D, $N_{xy}b^2/\pi^2D$, N_yb^2/π^2D, respectively
$\bar{R}_x^*, \bar{R}_{xy}^*, \bar{R}_y^*$	= $N_xb^2\cos^4\psi/\pi^2D$, $N_{xy}b^2\cos^3\psi/\pi^2D$, $N_yb^2\cos^2\psi/\pi^2D$, respectively
$W(x_1, y_1)$	= transverse deflection of the plate
x, y	= rectangular coordinates, see Fig. 1
x_1, y_1	= oblique coordinates, see Fig. 1
ψ	= angle of skew defined in Fig. 1
ν	= Poisson's ratio

Introduction

In this Note, a condensed version of Ref. 1, only the results are presented. The available results for buckling of clamped skew plates are few and far from complete.[2,3] In the present investigation, results for several new plate configurations and loading conditions as well as more accurate results for configurations reported in previous literature are obtained. In general, for a given a/b, the critical values increase with increasing skew angle. The results also confirm the conjecture of Ref. 4 that in the case of buckling under shear (N_{xy}), two critical values exist, the positive shear (one tending to reduce the skew angle) being numerically greater than the negative shear. However, reliable values for positive shear could not be obtained in Ref. 4 because of convergence difficulties. In

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